Int. J. Adv. Res. Sci. Technol. Volume 1, Issue1, Oct-2012, pp 10-13.

IJARST

International Journal of Advanced Research in Science and Technology

journal homepage: www.ijarst.com

ISSN 2319 – 1783

Number of Lattice Points

Shyam Sundar Agrawal*

Vikash College of Engineering for Women, Bargarh, Odisha, INDIA.

Corresponding Author's Email: dr.ssagrawal@rediffmail.com

A R T I C L E   I N F O

Article history:
Received 21 July 2012
Accepted 09 Sept. 2012
Available online 01 October 2012

Keywords:
Triangle,
Lattice points,
Pick's theorem.

A B S T R A C T

The present article is an attempt to illustrate the direct formula for calculating total number of points with integral co-ordinates (Lattice Points) inside a triangle and on the boundary of the triangle with vertices $(p,q),(p+|n|,q),(p,q+|n|)$ where p,q are integers and n is a real numbers.

Description:

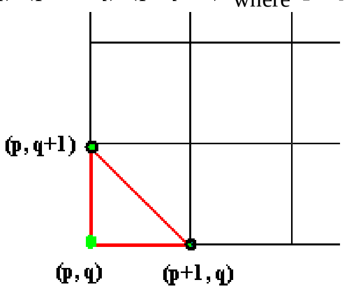
Lattice Point: It means the points in the plane with integer co-ordinates.

Let a triangle with vertices $(p,q),(p+|n|,q),(p,q+|n|)$ where p,q,n are integers.

I: Let n is a +ve integer

Case: 1:

Let a triangle with vertices $(p,q),(p+1,q),(p,q+1)$ where p,q are integers.

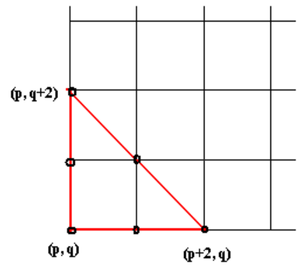


No. of Lattice points inside the triangle = 0

No. of Lattice points in the boundary of the triangle = 3 = 3 X 1.

Case: 2

Let a triangle with vertices $(p,q),(p+2,q),(p,q+2)$ where p,q are integers.

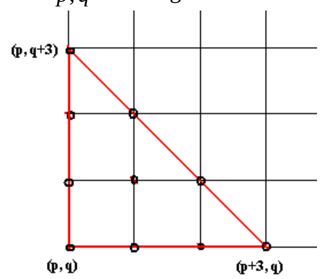


No. of Lattice points inside the triangle = 0

No. of Lattice points in the boundary of the triangle = 6 = 3 X 2.

Case: 3:

Let a triangle with vertices $(p,q),(p+3,q),(p,q+3)$ where p,q are integers.

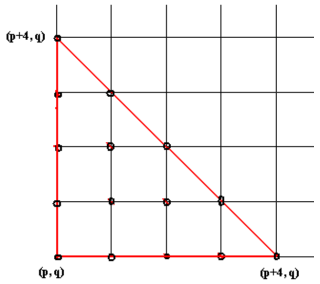


No. of Lattice points inside the triangle= 1=3-2

No. of Lattice points in the boundary of the triangle = 9.

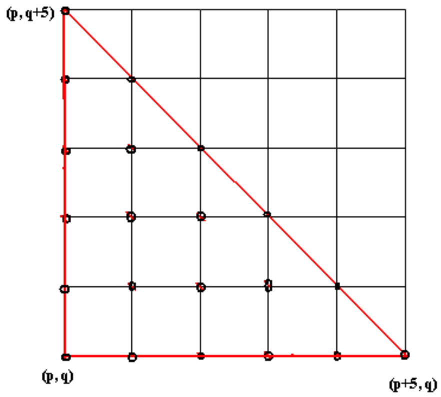
Case: 4:

Let a triangle with vertices $(p,q),(p+4,q),(p,q+4)$ where p,q are integers.



No. of Lattice points inside the triangle = 3 = (4-2) + (4-3)
No. of Lattice points in the boundary of the triangle = 12 = 3 X 4.

Case: 5:
Let a triangle with vertices $(p,q),(p+5,q),(p,q+5)$ where p,q are integers.



No. of Lattice points inside the triangle = (5-2) + (5-3) + (5-4)
No. of Lattice points in the boundary of the triangle = 15 = 3 X 5.

From above 5 cases we can conclude that if a triangle with vertices $(p,q),(p+n,q),(p,q+n)$ where p,q,n are integers and n is +ve integer, then
No. of Lattice points in the boundary of the triangle =

$$\begin{cases} 0 & n=0 \\ 3n & \text{n is a + ve integer}, n \geq 1 \end{cases}$$

No. of Lattice points inside the triangle =

$$= \begin{cases} 0 & n \leq 2 \\ (n-2)+(n-3)+.......+(n-(n-1)) & \text{n is a + ve integer}, n > 2 \end{cases}$$

$$= \begin{cases} 0 & n \leq 2 \\ \sum_{k=1}^{n-2} k = \dfrac{n^2-3n+2}{2} & \text{n is a + ve integer}, n > 2 \end{cases}$$

II: Let n is a negative integer:

As $|n| = |-n|$, so we can replace n by $|n|$

Hence Triangle having vertices $(p,q),(p+n,q),(p,q+n)$ where p,q,n are integers, then

No. of Lattice points in the boundary of the triangle =

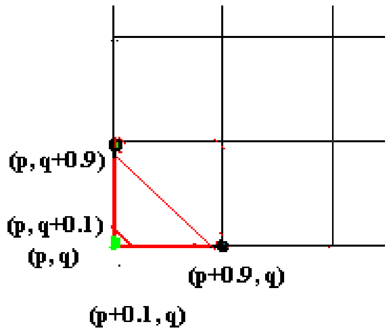
$$\begin{cases} 0 & n=0 \\ 3|n| & \text{n is a + ve integer}, |n| \geq 1 \end{cases}$$

No. of Lattice points inside the triangle

$$= \begin{cases} 0 & |n| \leq 2 \\ \sum_{k=1}^{|n|-2} k = \dfrac{|n|^2-3|n|+2}{2} & \text{n is a + ve integer}, |n| > 2 \end{cases}$$

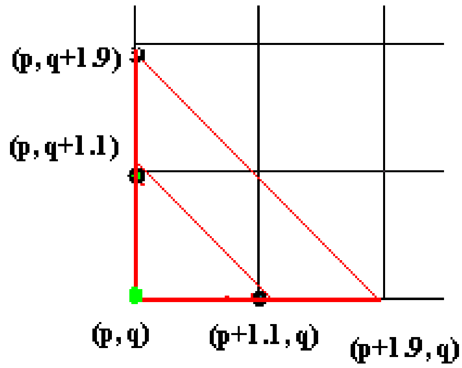
III: Let n is not an integer

Case 1: when 0<n<1



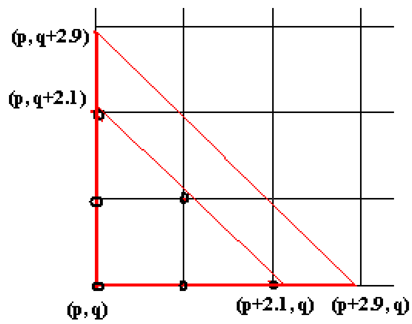
No. of Lattice points inside the triangle = 0
No. of Lattice points in the boundary of the triangle = 1= 2[n] + 1.

Case 2: when 1<n<2:



No. of Lattice points inside the triangle = 0
No. of Lattice points in the boundary of the triangle = 3 = 2[n]+1.

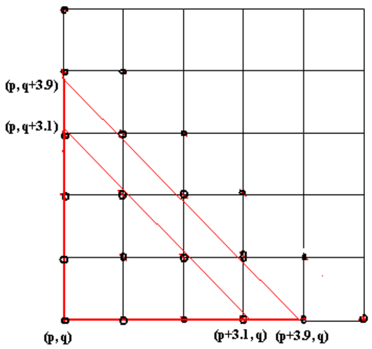
Case 3: when 2<n<3:



No. of Lattice points inside the triangle = 1 = [n] - 1

No. of Lattice points in the boundary of the triangle = 5 = 2[n]+1.

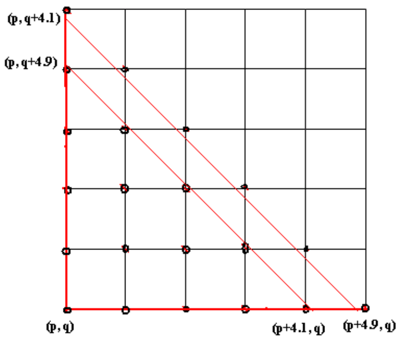
Case 4: when 3<n<4:



No. of Lattice points inside the triangle = ([n]-1) + ([n]-2)

No. of Lattice points in the boundary of the triangle = 7 = 2[n] + 1.

Case 5: when 4<n<5



No. of Lattice points inside the triangle

= ([n]-1) + ([n]-2) + ([n]-3)

No. of Lattice points in the boundary of the triangle = 9 = 2[n] + 1.

From above 4 cases we can conclude that if a triangle with vertices $(p,q),(p+n,q),(p,q+n)$ where $p,q,$ are integers and n is not an integer, then

No. of Lattice points in the boundary of the triangle =

$$\begin{cases} 0 & n=0 \\ 2[n]+1 & \text{n is a + ve integer}, n \geq 0 \end{cases}$$

No. of Lattice points inside the triangle =

$$= \begin{cases} 0 & n \leq 2 \\ \sum_{k=1}^{[n]-1} k = \frac{[n]^2 - [n]}{2} & \text{n is a + ve integer}, n > 2 \end{cases}$$

Form the conclusion of I, II and III we can conclude that

If a triangle with vertices $(p,q),(p+n,q),(p,q+n)$ where $p,q,$ are integers and n is real number, then

Number of Lattice points inside the triangle=

$$\begin{cases} 0 & |n| \leq 2 \\ \sum_{k=1}^{|n|-2} k = \frac{(|n|^2 - 3|n| + 2)}{2} & \text{n is an integer}, |n| > 2 \\ \sum_{k=1}^{[|n|]-1} k = \frac{([|n|]^2 - [|n|])}{2} & \text{n is not an integer}, |n| > 2 \end{cases}$$

Number of the Lattice points on the boundary of the triangle=

$$\begin{cases} 0 & \text{n is not an integer}, |n| < 1 \\ 3|n| & \text{n is an integer}, |n| \geq 1 \\ 2[|n|]+1 & \text{n is not an integer}, |n| > 0 \end{cases}$$

The above result also satisfies Pick's Theorem.

Proof: - Let a triangle with vertices $(p,q),(p+n,q),(p,q+n)$ where p,q and n are integers.

Then Area of triangle = A = $\frac{n^2}{2}$

No. of Lattice point inside the triangle = I

$$I = \begin{cases} 0 & |n| < 2 \\ \sum_{k=1}^{|n|-2} k & \text{n is an integer}, |n| \geq 2 \end{cases}$$

No. of Lattice points on the boundary of the triangle = B

$$B = \{3|n| \quad \text{n is an integer}, |n| \geq 1$$

Case 1: When n = 1.

A = $\frac{n^2}{2} = \frac{1}{2}$.

I = 0

B = 3

Now, I + B/2 – 1 = 0 + 3/2 – 1 = ½ =A

Which satisfies Pick's theorem

A = I + B/2 – 1.

Case 2: When n = 2.

A = $\frac{n^2}{2} = 2$., I = 0, B = 6

Now, I + B/2 – 1 = 0 + 3 – 1 = 2 = A

Which satisfies Pick's theorem

A = I + B/2 – 1.

Case 3: - when $n > 2$ A = $\frac{n^2}{2}$,

$$I = \sum_{k=1}^{|n|-2} k = 1+2+3+......+(n-2) = \frac{(n-2)(n-1)}{2},$$

B = $3n$

A= I + B/2 – 1 =

$$\frac{(n-2)(n-1)}{2} + \frac{3n}{2} - 1 = \frac{(n-2)(n-1)+3n-2}{2} = \frac{n^2-3n+2+3n-2}{2} = \frac{n^2}{2}$$

So, from Case 1, 2 and 3 it satisfies Pick's theorem.

Conclusion

Based on our informal findings, it appears as though one can find directly the total number of points with integral co-ordinates (Lattice Points) inside a triangle and on the boundary of the triangle with vertices $(p,q),(p+n,q),(p,q+n)$ where p,q are integers and n is a real numbers. I have not conclusively proved our conjecture, but instead leave this as an exercise to the interested readers.

References

[1] Posamentier, A. (2002). Advanced Euclidean Geometry. Emeryville, CA: Key College Press.

[2] Agrawal, Shyam Sundar (2009). "Counting Similar Triangles", *Ohio Journal of School Mathematics*, 60 PP 13 – 18.

[3] Clements, G. F. (Jan., 1971). "Sets of Lattice Points which Contain a Maximal Number of Edges", *Proceedings of the American Mathematical Society*, Vol. 27, No. 1, pp. 13-15.

[4] Arkhipova, L. G. (2008). "Number of Lattice points in a sphere", Moscow University Mathematics Bulletin, Vol. 63, No. 5, pp. 59-61.

[5] Hammer, J(1977). "Unsolved Problems Concerning Lattice Points", Pitman, London.